Linear Algebra MTH 221 Spring 2011, 1-3

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**Exam II**, MTH 221, Spring 2011

QUESTION 1. (POINTS) For each question below, circle the right answer.

(i) One of the following is a subspace of  $R^3$ :

a. 
$$S = \{(a+b, -2a, b+1) \mid a, b \in R\}^{\times}$$

b. 
$$S = \{(a-2b, b^2-a, 0) \mid a, b \in R\} \times$$

c. 
$$S = \{(3a, 2b + a, 5) \mid a, b \in R\} \times$$

(d) 
$$S = \{(0,0,3a+2b) \mid a,b \in R\}$$

(ii) One of the following is a subspace of  $P_2$ :

a. 
$$S = \{(a+b) + 2x \mid a, b \in R\}$$
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b. 
$$S = \{a + bx + (a - b)x^2 \mid a, b \in R\} \times$$

c. 
$$S = \{3 + ax \mid a \in R\} \times$$

$$\widehat{\mathbf{d.}} S = \{a + 3ax \mid a \in R\}$$

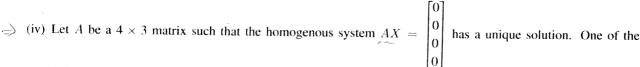
(iii) One of the following is a linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^3$ :

a. 
$$T: \mathbb{R}^3 \to \mathbb{R}^3, T(a, b, c) = (0, 1, a + b + c)$$

b. 
$$T: \mathbb{R}^3 \to \mathbb{R}^3, T(a,b,c) = (3a+b^2,c,0)$$

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$$T: \mathbb{R}^3 \to \mathbb{R}^3, T(a,b,c) = (a+b,0,c-2b)$$

d. 
$$T: \mathbb{R}^3 \to \mathbb{R}^3, T(a, b, c) = (3a, -b, 2 + c) \times$$



following statements is correct:

- a. The rows of A are independents.
- b. Rank(A) is at most 2.
- The columns of A are independent
- d. Exactly one column of A is a linear combination of the other two columns.
- (v) If  $T: \mathbb{R}^5 \to \mathbb{R}^3$  is a linear transformation such that dim(Ker(T)) = 3, then one of the following statement is correct:
  - a. The range of T equals to  $\mathbb{R}^3$ .
  - **6** The range of T "lives" inside  $R^3$  but not equal to  $R^3$
  - c. It is possible to find three independent points in the range of T.
  - d. If A, B are independent points in  $R^3$ , then Range of  $T = Span\{A, B\}$ .
  - e. (a) and (c)
- (vi) Let  $T: P_2 \to R$  be a linear transformation such that T(2+x) = 4, T(-1-x) = -4. Then T(2) = -4.
  - a. 2.
  - (b) 0.
  - c. -2
  - d. None of the above.

(vii) Given 
$$S = \begin{bmatrix} a+b & 0 & -a-b \\ c & 0 & -c \end{bmatrix} \mid a,b,c \in R \}$$
 is a subspace of  $R^{2\times 3}$ . Then  $dim(S) = 1$ 

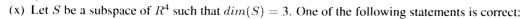
- (a) 2
- b. 3
- c. 6
- d. None of the above.

(viii) Let 
$$S = span\{(1, -1, 0), (2, -1, 0), (1, 0, 0)\}$$
. Then  $dim(S) =$ 

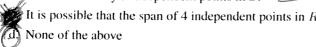
- ъ. з
- c. 1
- d. None of the above

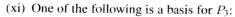
$$\Rightarrow$$
 (ix) Let  $S = span\{(1,1,0), (1,-1,1), (0,-2,1)\}$ . Then

- a. Every point in  $\mathbb{R}^3$  belongs to S.
- b. dim(S) = 3
- c. (a) and (b) are correct
- (d.) The point (4, -2, 0) does not belong to S.
- None of of the above



- a. Every 3 points in S form a basis for S.
- Connect answer (1),
- b. There are exactly 3 independent points in S. L. You circle B It is possible that the span of 4 independent points in  $\mathbb{R}^4$  equals to S





a. 
$$B = \{1 + x^2, -x + x^2, 3, x\}$$

b. 
$$B = \{1 + x, 2, -2x\}$$

C. 
$$B = \{1 + x + x^2, x + x^2, 1 + x^2\}$$
  
d.  $B = \{1 + x, 3 + x + x^2, 2 + x^2\}$ 

$$\overrightarrow{d}$$
.  $B = \{1 + x, 3 + x + x^2, 2 + x^2\}$ 

(xii) Let A be a 
$$4 \times 4$$
 matrix such that  $det(A) = 3$ . Then one of the following statements is correct:

- a. Rank(A) is at most 3. ≿
- b. span of the columns of A "lives" in  $R^4$  but not equal  $R^4$ . x
- $\Re \operatorname{Rank}(A) = 4$ .
- d. At least one column of A is a linear combination of the other three columns.  $\chi$

(xiii) Let 
$$D = \{a + (b+a)x + 3bx^2 \mid a, b \in R\}$$
 be a subspace of  $P_3$ . Then a basis for  $D$  is:

a. 
$$B = \{1 + x, x^2\}$$

(b) 
$$B = \{1 + x, x + 3x^2\}$$

c. 
$$B = \{x + 3x^2, 1 + 2x\}$$

- d. All the above.
- (xiv) Let  $T: \mathbb{R}^2 \to \mathbb{R}$  be a linear transformation such that T(1,0) = 4, T(2,-2) = 2. Then the standard matrix representation of T is:

d. None of the above

- $\begin{array}{l} \text{span}\{(-1,1,0,0,0),(-1,0,1,0,0),(-1,0,0,1,0)\} \\ \text{b. span}\{(-1,1,1,1,0)\} \end{array}$
- c. span $\{(1, -1, -1, 0, 0), (1, 0, 0, -1, 0), (-1, 0, 0, 0, 1)\}$
- d. span  $\{(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0)\}$  ×

None of the above

- (xvi) Let A be the above matrix. Then Column(A) =
  - (a)  $Span\{(1,-1,2),(1,0,-2)\}$
  - b.  $Span\{(1,0,0),(1,1,0)\}$
  - c. Span $\{(1,0,0),(1,1,-4)\}$
  - d. All the above are correct.
- (xvii) Let  $T: P_3 \to R$  such that  $T(a+bx+cx^2) = \int_0^1 (a+bx+cx^2) dx$ . Then  $ker(T) = \int_0^1 (a+bx+cx^2) dx$ .
  - a. Span $\{1, x, x^2\}$
  - b. Span $\{-0.5 + x\}$
  - c. Span  $\{\frac{-2.5}{3} + x + x^2\}$
  - (d) Span  $\{-0.5 + x, \frac{-1}{3} + x^2\}$
  - e. None of the above
- (xviii) Let  $T = R^4 \to R^3$  such that T(a, b, c, d) = (a + b + c 2d, -2d, d). We know T is a linear transformation. The standard matrix representation of T is

a. 
$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \end{bmatrix}$$

b. 
$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$c. \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c|cccc}
\hline
\mathbf{d.} & \begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{array}$$

- (xix) In the Previous Question, dim(Ker(T)) =
  - a. 1
  - b. 3
  - (c) 2
- (xx) In Question number 18 (XVIII). range(T) =
  - a. Span  $\{(1, 1, 1, -2), (0, 0, 0, -2)\}$
  - b. Span  $\{(1,0,0,-2)\}$
  - (c) Span  $\{(1,0,0),(-2,-2,1)\}$
  - d. Span ((1,0,0),(0,-2,0))
  - e. None of the above

## **Faculty information**